Problem Set 4
Statistical Mechanics
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Problem 9: Relations between material parameters

Show the following equations:

(a)

$$\kappa_S = \kappa_T - TV\frac{\alpha_P^2}{C_P}$$

Here $\kappa_S = -V^{-1}\partial V/\partial P|_S$ is the adiabatic compressibility.

(3 points)

(b)

$$\frac{\kappa_S}{\kappa_T} = \frac{C_V}{C_P}$$

(3 points)

Problem 10: Stability - completion of derivation

In lecture 7 our goal was the formula

$$\Delta S = \frac{1}{2T}\left(-\frac{C_V}{T}\Delta T^2 - \frac{1}{V\kappa_T}\Delta V_n^2 - \frac{\partial \mu}{\partial n}\Big|_{T,P}\Delta n^2\right) \ . \tag{1}$$

The quantity ΔS is the entropy reduction in response to fluctuations of temperature, ΔT, volume (at constant n), ΔV and particle number Δn. However, in class we made it to

$$\Delta S = \frac{1}{2T}\sum_\nu \left(-\frac{C_V}{T}\Delta T_\nu^2 - \frac{1}{V\kappa_T}\Delta V_\nu^2 - \frac{\partial \mu_\nu}{\partial n_\nu}\Big|^o_{T_\nu,V_\nu}\Delta n_\nu^2 - 2\frac{\partial \mu_\nu}{\partial V_\nu}\Big|^o_{T_\nu,n_\nu}\Delta n_\nu \Delta V_\nu\right) \ .$$

only. Using

$$\Delta V_\nu = \underbrace{\frac{\partial V_\nu}{\partial T_\nu}\Big|^o_{P_\nu,n_\nu}\Delta T_\nu + \frac{\partial V_\nu}{\partial P_\nu}\Big|^o_{T_\nu,n_\nu}\Delta P_\nu}_{=\Delta V_{n,\nu}} + \frac{\partial V_\nu}{\partial n_\nu}\Big|^o_{T_\nu,P_\nu}\Delta n_\nu$$

show that Eq. (1) follows (note: $\Delta x^2 = \sum_\nu \Delta x_\nu^2$).

(6 points)

Problem 11: Phase boundary and phase change enthalpy

Steam ironing stations are offered on the Internet with the following specifications: Power iron 2200 W, steam pressure 5 bar, 120 g/min continuous steam output. We would like to check these specifications for consistency.

First, determine the temperature of the saturated steam when its pressure is 5 bar as specified - that is, T_v(5 bar). For this purpose, you can use the following source, for example - Handbook of Chemistry and Physics, D.R. Lide (Editor), CRC Press (Bib-signature 74 STM1139-98); look for 'vapor pressure (for water)'.

In class we had derived the Clapeyron equation in the form

$$\left.\frac{dP}{dT}\right|_{coex} = \frac{\Delta H}{T\Delta V} \ .$$

The subscript *coex* means that the derivative is along the phase boundary. Δ refers to the change of the respective quantity, i.e. enthalpy or volume, during a phase transformation (e.g. water to steam or vice versa). From the source given above, at T_v(5 bar) you can estimate the slope $dP/dT|_{coex}$. The above formula then gives you the enthalpy of vaporization ΔH_v at T_v(5 bar) (assuming that $\Delta V = V_{\text{vapor}} - V_{\text{liquid water}} \approx V_{\text{ideal steam}}$). Again, compare your number (in kJ/mol) with the corresponding number from the source (search for the term 'enthalpy of vaporization (water)').

Now calculate how many grams of water you can convert to steam per minute at the given conditions with 2200 W. Not considered is the power you need to heat the original tap water from e.g. 20 ^{o}C to T_v(5 bar). How large would this be?

(6 points)