Problem Set 2
Statistical Mechanics
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Problem 2: Volume work

A compressor adiabatically compresses a gas, which originally is at room temperature T_o and ambient pressure P_o. After the gas has passed through a water cooled system of pipes (isobaric cooling), it leaves the machine again possessing the temperature T_o but with the pressure P_1.

(a) Calculate the necessary work for this process w_a. Also calculate the work w_b in the case of a reversible, isothermal compression yielding the identical final state. Sketch the ratio w_a/w_b as function of P_1/P_o with $C_V = (5/2)nR$. Hint: Make a sketch of the processes in term of suitable thermodynamic variables. Assume an ideal gas. For additional useful information check the sections *Isotherms and adiabatic curves* and *Efficiency of engines with ideal gas as working substance* in *Thermodynamics*.

(6 points)

(b) Discuss the gas entropy change for the two processes. Note that there is a fast answer as well as the alternative of an explicit calculation. You should provide both. Hint: $\partial E/\partial V|_T = 0$ for an ideal gas.

(6 points)

Problem 3: Heat

Consider a perfectly insulated, closed cylinder containing an ideal gas, possessing the pressure $P_1 = 1$bar and the temperature $T_1 = 20^o$C. Initially the cylinder and the gas is divided in two equal volumes by a frictionless, ideally insulating piston. Each volume contains 1 mol of gas. The volume occupied by the piston is negligible. In a one of the two compartments there is an electric heating coil. After a current has passed through the coil the gas temperature in this compartment has risen to $T_2 = 300^o$C. What is the amount of heat, ΔQ (in Joule), which has flown into the system? Hint: Again make use of $\partial E/\partial V|_T = 0$ and $C_V = (5/2)nR$.

(6 points)

Problem 4: Ideal Gas Energy

We have seen that the entropy of an ideal gas changes according to

$$S(T, V) - S(T_o, V_o) = nR \ln \left((T/T_o)^{3/2} (V/V_o) \right) \ .$$

If the ideal gas is compressed adiabatically (no heat is exchanged with the outside, which here means $\Delta S = 0$) the equation tells us that the temperature rises ($T > T_o$) and therefore its internal energy E increases. Can you give a (qualitative) physical explanation for this phenomenon, i.e. how is energy transferred to the gas particles?

(3 points)